NYQUIST PLOT

PRINCIPLE OF ARGUMENT

Principle of argument states that if there are P poles and Z zeros are enclosed by the 's' plane closed path, then the corresponding $G(s) \cdot H(s)$ plane must encircle the origin P-Z times.

Number of encirclements

$$N = P - Z.$$

If the enclosed 's' plane close path contains only poles, then the direction of the encirclement in the $G(s) \cdot H(s)$ plane will be opposite to the direction of the closed path in the 's' plane.

If the enclosed 's' plane close path contains only zeros, then the direction of the encirclement in the $G(s) \cdot H(s)$ plane will be in the same direction as that of enclosed path in the 's' plane.

fig 9.3 fig 9.4

NYQUIST STABILITY CRITERION.

Let us now apply the principle of argument to the entire right half of 's' plane by selecting it as a closed path.

This selecting path is called the Nyquist Contour.

We know that the closed loop control system is stable if all the poles of the closed loop transfer function are in the left half on 's' plane.

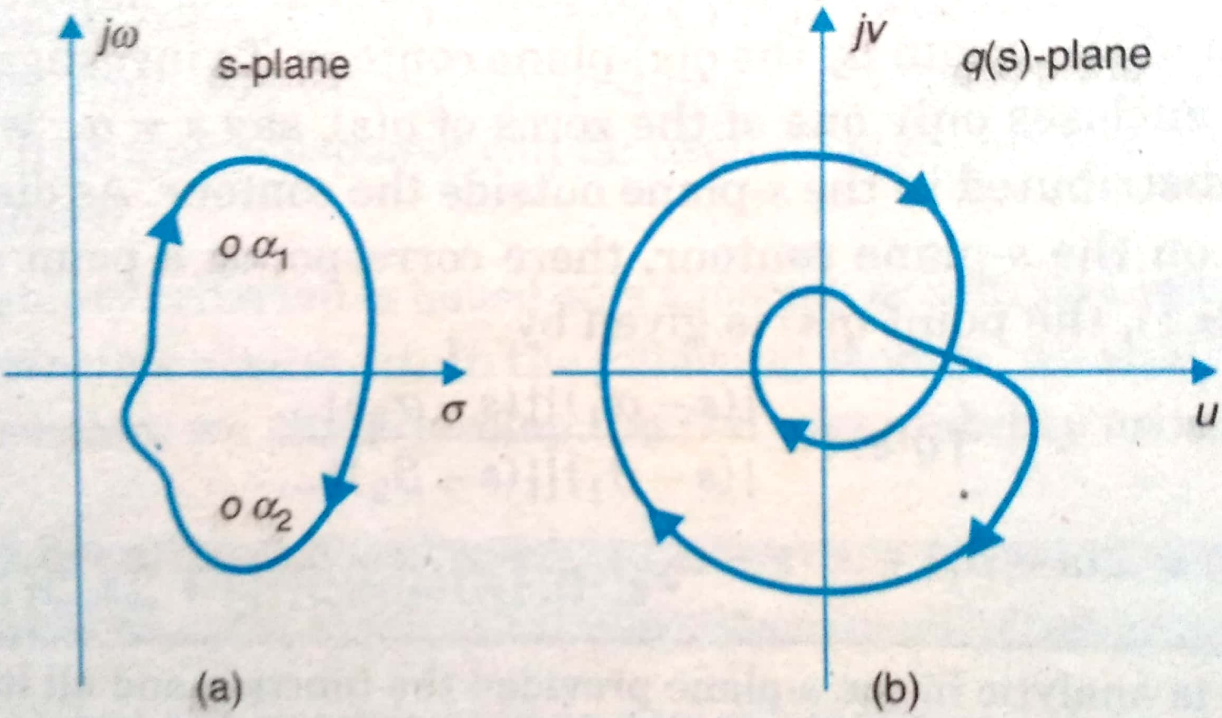

Fig. 9.3. An s-plane contour enclosing two zeros of q(s) and the corresponding q(s)-plane contour.

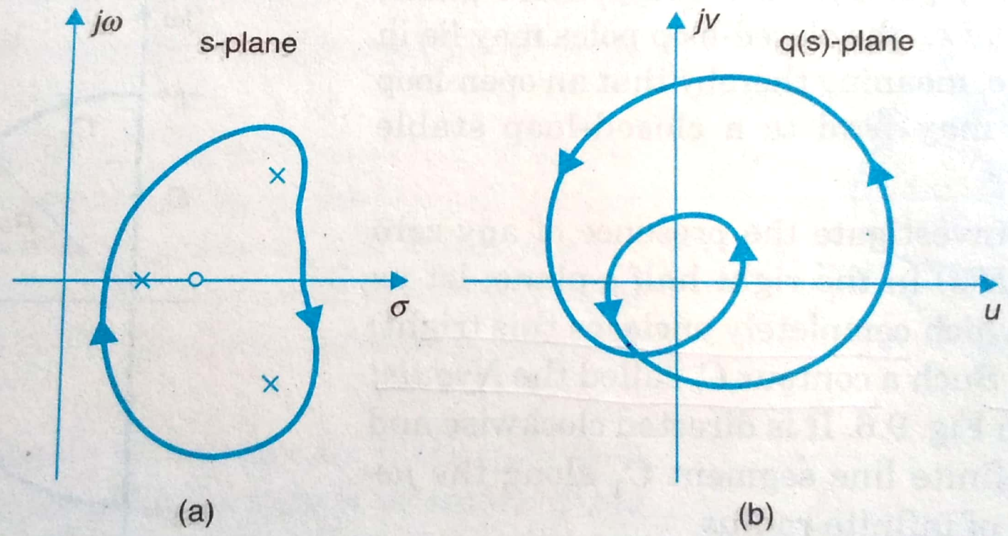

Fig. 9.4. Mapping of the s-plane contour which encloses 1 zero and 3 poles.

poles of the closed loop transfer function are the nothing but the roots of the characteristic equ^n.
i.e $1+G(s) \cdot H(s) = 0$,
As the order of the characteristic equ^n increases it is difficult to find the roots.

The poles of the characteristic equ^n $(1+G(s) \cdot H(s) = 0)$ are same as that of the poles of the open loop transfer function $(G(s) \cdot H(s))$.

The zeros of the characteristic equ^n $(1+G(s) \cdot H(s) = 0)$ are same as that of the zeros of the closed loop transfer function.

In order for the system to be stable there should be no zeros of $q(s) = 1+G(s) \cdot H(s)$ on the right half s-plane.

$$Z = 0$$

$$N = P$$

In special case of $P = 0$ (i.e the open loop stable system) the closed loop system is stable if

$$N = P = 0.$$

Which means the net encirclements of the origin of the $q(s)$ plane by the Γ_q contour should be zero.

$$G(s) \cdot H(s) = [1+G(s) \cdot H(s)] - 1$$

Γ_{GH} contour of $G(s) H(s)$. Corresponding to Nyquist contour in the s-plane is the same as contour Γ_q of $1+G(s) \cdot H(s)$ drawn for the point $-1+j0$.

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		1	2	3	1	2
3	7	8	9	10	8	9
10	14	15	16	17	15	16
17	21	22	23	24	22	23
24	28	29	30	31	29	30

Encirclement of the origin by the contour Γ_q is equivalent to the encirclement of the point $(-1+j0)$ by the contour Γ_{GH}.

Fig 9.6. Fig 9.7

Along C_1

$s = j\omega$ with s varying from $-j\infty$ to $+j\infty$.

and along C_2

$s = \lim_{R\to\infty} Re^{j\theta}$ with θ varying from $+\pi/2$ to 0 to $-\pi/2$

Statement of Nyquist Stability criterion.

If the contour Γ_{GH} of the open loop transfer function $G(s)H(s)$ corresponding to the Nyquist Contour in the s-plane encircles the point $(-1+j0)$ in the counter clockwise dir^n as many as times as the number of right half s-plane poles of $G(s)H(s)$, the closed loop system is stable.

Sunday 01

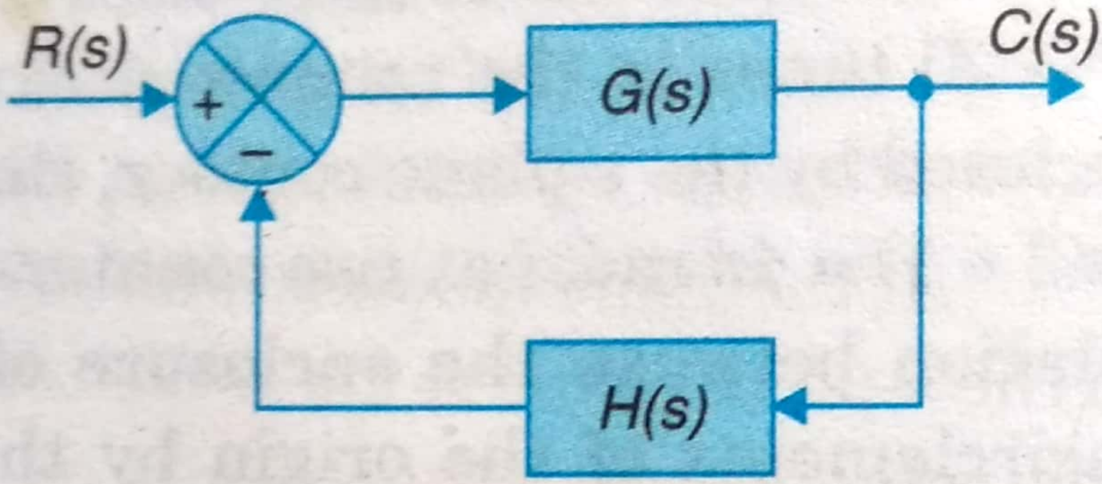

Fig. 9.5. A feedback control system.

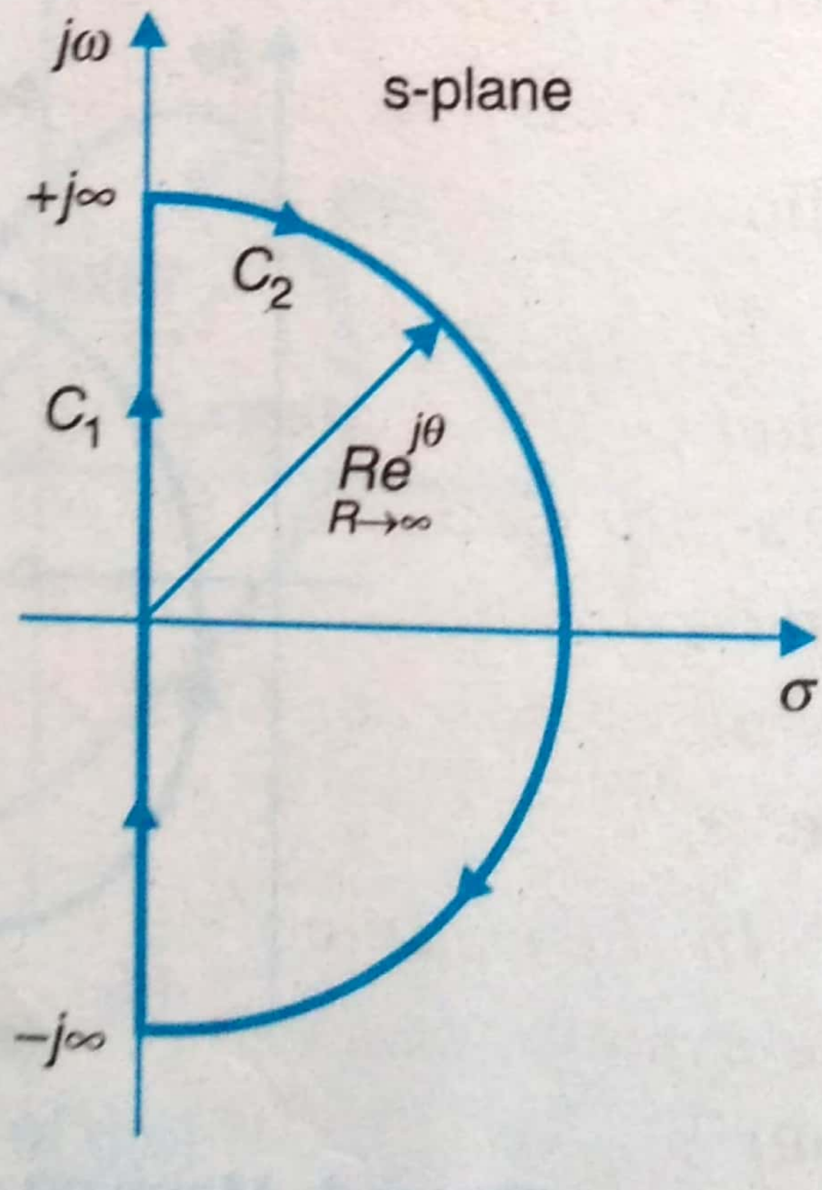

Fig. 9.6. The Nyquist contour.

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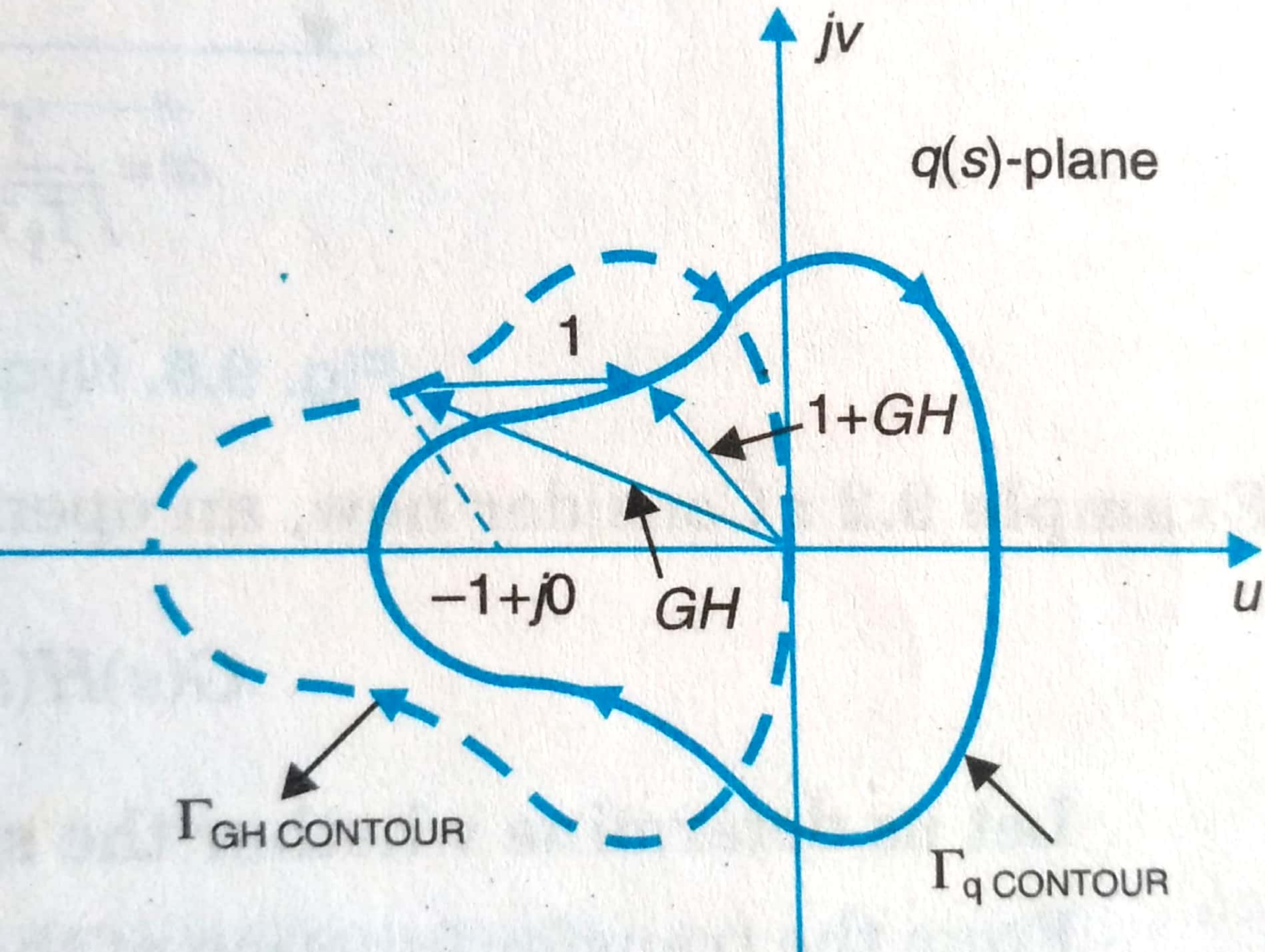

Fig. 9.7

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Ilustrative Example 1. *Examine the closed-loop stability of a control system whose open-loop transfer function is given below :*

$$G(s)H(s) = \frac{K}{s(sT+1)}.$$

Solution. $$G(s)H(s) = \frac{K}{s(sT+1)}$$

Put $$s = j\omega$$

$$\therefore \quad G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega T+1)} \quad ...(1)$$

Rationalizing Eq. (1) and separating into real and imaginary parts following equation is obtained

$$G(j\omega)H(j\omega) = -\frac{KT}{(\omega^2T^2+1)} - \frac{jK}{\omega(\omega^2T^2+1)} \quad ...(2)$$

From Eq. (2) it is observed that as ω increases from $\omega = +0$ to $\omega = +\infty$ both the real part and the imaginary part lie in the third quadrant of $G(s)H(s)$-plane.

At $\omega = +0$, $\angle G(+j0)H(+j0) = -90°$ and the magnitude approaches infinity.

At $\omega = +\infty$, $\angle G(+j\infty)H(+j\infty) = -180°$ and the magnitude approaches zero.

The Nyquist plot as ω varied from $\omega = +0$ to $\omega = +\infty$ is shown in Fig. 7.6.7. The plot for $\omega = -\infty$ to $\omega = -0$ is shown by dotted lines.

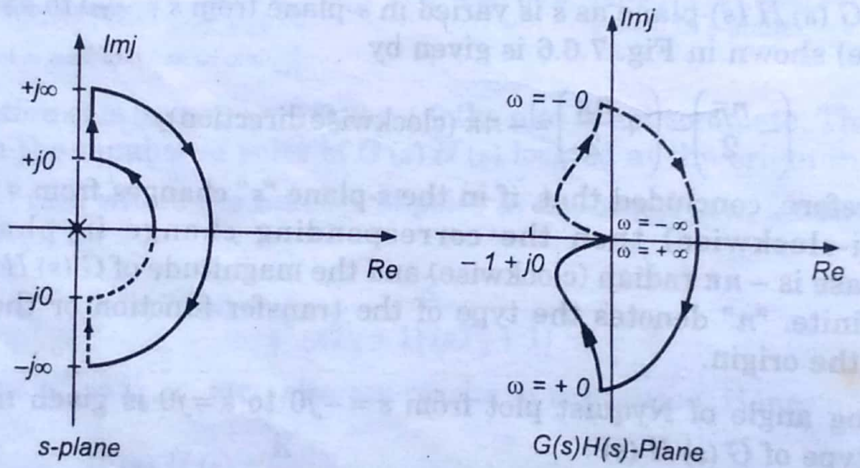

Fig. 7.6.7. Nyquist plot for $G(s)H(s) = \frac{K}{s(sT+1)}$.

As the system is type 1 the plot is closed from $\omega = -0$ to $\omega = +0$ through an angle of $-\pi$ (clockwise) with an infinite radius. The arrowheads shown in Fig. 7.6.7 are in the direction of increasing ω.

As the point $(-1+j0)$ is not encircled by the plot, therefore,

$$N = 0$$

The number of zeros (roots) of the characteristic equation with positive real part is determined by using relation*

$$N = (P_+ - Z_+)$$

because, $$P_+ = 0 \qquad \therefore \quad Z_+ = 0$$

Hence, the number of zeros (roots of the characteristics equation) with positive real part is nil and the closed-loop system is stable.

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RULES FOR DRAWING NYQUIST PLOTS

1) Locate the poles and zeros of the open loop transfer function $G(s).H(s)$. in S-plane.

2) Draw the polar plot varying ω from 0 to ∞. if the poles or zero present at $s=0$.

3) Draw the mirror image of above polar plot for values of ω ranging from $-\infty$ to 0^-.

4) The number of infinite radius half circles will be equal to the number of poles or zeros at origin.

The infinite radius half circle will start at the point where the mirror image of the polar plot ends.

And the infinite radius half circle will end at the point where the polar plot starts.

After drawing the Nyquist plot, we can find the stability of the closed loop control system using the Nyquist stability criterion

If the critical point $(-1+j0)$ lies outside the encirclement, then the close loop control system is absolute stable.

STABILITY ANALYSIS USING NYQUIST PLOT.

From the Nyquist plots, we can identify whether the control system is stable, marginally stable unstable based on parameter,

(i) Gain cross over frequency (ω_{gc}) and phase cross over frequency (ω_{pc})

(ii) Gain Margin and phase Margin.

phase cross over frequency (ω_{pc})

The frequency at which the Nyquist plot intersects the negative real axis (phase angle is 180°) is known as phase cross over frequency (ω_{pc}).

Gain crossover frequency (ω_{gc})

The frequency at which the Nyquist plot is having the magnitude of one is known as ~~gain~~ gain cross over frequency (ω_{gc}).

For stable system $\omega_{pc} > \omega_{gc}$.

For Marginally stable system $\omega_{pc} = \omega_{gc}$.

For unstable system $\omega_{pc} < \omega_{gc}$.

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Gain Margin: (GM)

The gain margin GM is equal to the reciprocal of the Magnitude of the Nyquist plot at the phase cross over frequency.

$$G.M = \frac{1}{M_{pc}}.$$

Where M_{pc} is the magnitude at phase cross over frequency in normal scale.

Phase Margin: (PM)

The phase margin (PM) is equal to the sum of 180° and the phase angle at the gain cross over frequency.

$$PM = 180^\circ + \phi_{gc}.$$

Where ϕ_{gc} is the phase angle at gain cross over frequency

For stable system $GM > 1$, PM is +ve.

For marginally stable system $GM = 1$, $PM = 0$ degree

For unstable system $GM < 1$, PM is −ve.

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8	9	10	11	12	13	14

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NYQUIST STABILITY CRITERION APPLIED TO INVERSE POLAR PLOT.

Occasionally, it is found more convenient to work with the inverse function $\frac{1}{G(j\omega)H(j\omega)}$ rather than the direct function $G(j\omega)H(j\omega)$.

Nyquist stability criterion can be applied to inverse polar plot from direct polar plot after minor modification.

Let us consider a open-loop transfer function.

$$G(s)H(s) = K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} \; ; \; m \leq n. \quad \text{--- (1)}$$

For stable system, no roots of the characteristic eqn should lie in the right half of s-plane.

$$q(s) = 1 + G(s)H(s) = \frac{(s+z_1')(s+z_2')\cdots(s+z_n')}{(s+P_1)(s+P_2)\cdots(s+P_n)} \quad \text{--- (2)}$$

Dividing eqn (2) by eqn (1) we get

$$q'(s) = \frac{1}{G(s)H(s)} + 1 = \frac{(s+z_1')(s+z_2')\cdots(s+z_n')}{(s+z_1)(s+z_2)\cdots(s+z_m)} \quad \text{--- (3)}$$

From eqn (2) & (3), it is found that

(i) Zeros of $q(s)$ & $q'(s)$ are same.

(ii) poles of $q(s)$ and $G(s)\cdot H(s)$ are same

(iii) poles of $q'(s)$ and $\frac{1}{G(s)\cdot H(s)}$ are same.

and also same with zeros of $G(s)\cdot H(s)$.

If $\frac{1}{G(s)H(s)}$ has P right half s-plane poles and characteristic eqn has Z right half s-plane zeros

The Locus $\frac{1}{G(s)H(s)}$ encircles the point $(-1+j0)$ N-times in counter clockwise dirn.

$$N = P - Z.$$

For stability $Z = 0$,

so, $N = P$.

If the Nyquist plot $\frac{1}{G(s) \cdot H(s)}$, corresponding to the Nyquist Contour in the s-plane encircles $(-1+j0)$ in counter clockwise as many a the right half s-plane poles of $\frac{1}{G(s) \cdot H(s)}$.

Then the close loop system is stable.

Special case of no poles on right half s-plane of $\frac{1}{G(s)H(s)}$

$N = 0$, for stable system.

Example 9.9 : Consider a feedback system with an open-loop transfer function

$$G(s)H(s) = K/s(Ts + 1)$$

The inverse polar plot of $G(s)H(s)$ corresponding to the s-plane Nyquist contour of Fig. 9.18(a) is obtained in steps below.

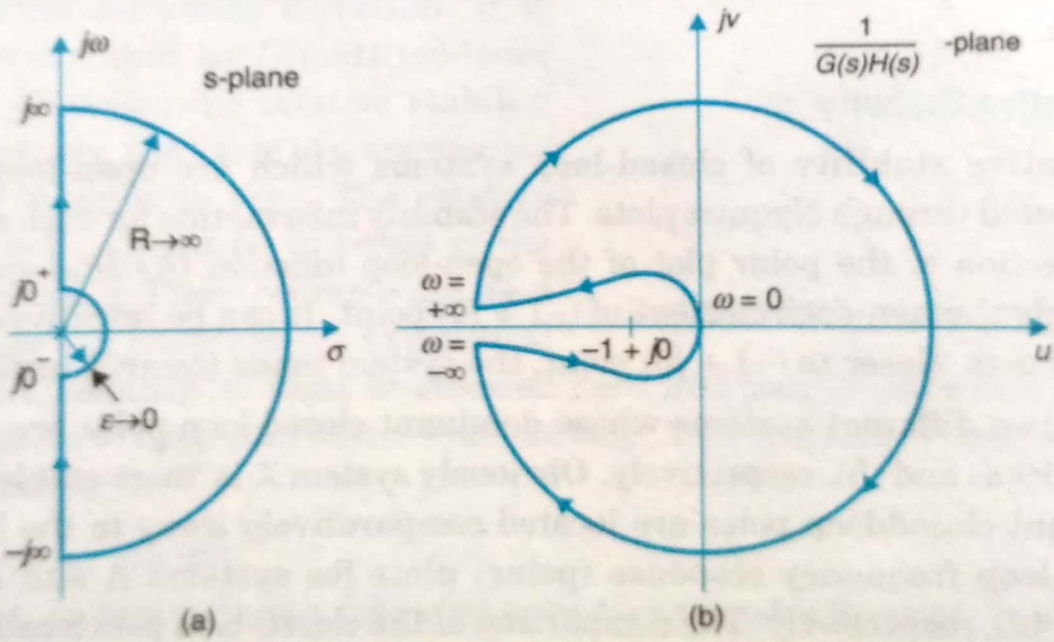

Fig. 9.18. The Nyquist contour and the corresponding plot of 1/G(s)H(s) = s(sT + 1)/K.

1. The semicircular indent around the origin in the s-plane is represented by

$$s = \lim_{\varepsilon \to 0} \varepsilon e^{j\theta}; \text{ where } \theta \text{ varies from } -90^\circ \text{ through } 0^\circ \text{ to } +90^\circ.$$

It is mapped into $1/G(s)H(s)$-plane as

$$\lim_{\varepsilon \to 0} \left[\frac{\varepsilon e^{j\theta}(\varepsilon e^{j\theta}T + 1)}{K} \right] = \lim_{\varepsilon \to 0} \frac{\varepsilon}{K} e^{j\theta} = 0e^{j}\theta$$

2. Along the $j\omega$-axis $1/G(j\omega)H(j\omega) = j\omega(j\omega T + 1)/K$.

3. The infinite semicircle in the s-plane represented by

$$s = \lim_{R \to \infty} Re^{j\theta};\ \theta \text{ varies from } + 90^\circ \text{ through } 0^\circ \text{ to } -90^\circ$$

is mapped into the $1/G(s)H(s)$-plane as

$$\lim_{R \to \infty} \frac{Re^{j\theta}(Re^{j\theta} + 1)}{K} = \lim_{R \to \infty} \frac{R^2}{K} e^{j2\theta}$$

which is a circle of infinite radius with angle varying from 180° through 0° to −180°.

The inverse polar plot of $G(s)H(s)$ obtained from the above steps is shown in Fig. 9.18(b). It is found that $(-1 + j0)$ point is not encircled by $1/G(s)H(s)$-locus. Further since $1/G(s)H(s) = s(Ts + 1)/K$ has no poles in the right half s-plane, the system is stable.

7.8. RELATIVE STABILITY FROM NYQUIST PLOT

Fig. 7.8.1 shows Nyquist plots for two systems which are stable. As both the plots are crossing the negative real axis at same point, the two systems have the same gain margin. However, the two systems have different phase margin. The system B has more phase margin than system A. Therefore, system B is relatively more stable than system A.

Similarly, if two systems have same phase margin but different gain margin then the system having greater gain margin is relatively more stable than the system with lesser gain margin.

Conditionally stable systems. In the Nyquist plot shown in Fig. 7.8.2 the location of the point $(-1+j0)$ depends on the value of forward path gain K. For smaller range of K the point $(-1+j0)$ lies between oa, any increase in the value of K beyond this range brings the point $(-1+j0)$ between ab and if, K is further increased then the point $(-1+j0)$ lies between bc.

It is given that the open-loop transfer function of the system has the number of poles with positive real part as nil, therefore, if the point $(-1+j0)$ lies between oa or bc the number of encirclements of the point $(-1+j0)$ by the Nyquist plot are -2 indicating that the closed-loop system is unstable.

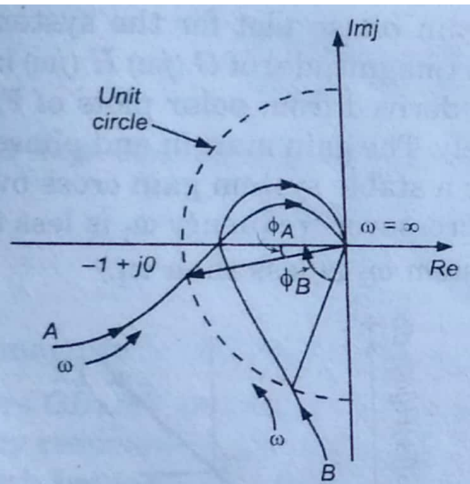

Fig. 7.8.1. Nyquist plot for two systems having same gain margin but different phase margin.

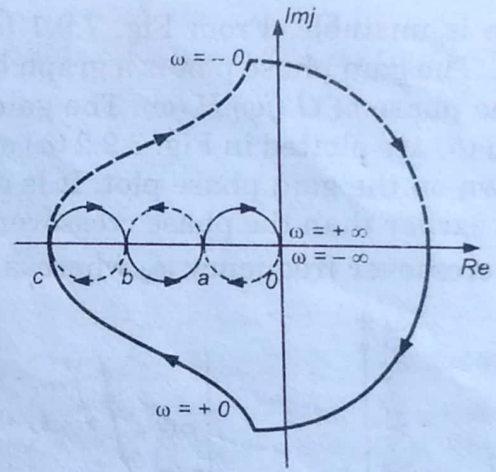

Fig. 7.8.2. Nyquist plot of conditionally stable system.

However, if the point $(-1 + j0)$ lies between ab the point $(-1 + j0)$ is encircled once in clockwise direction and then once again in the anti-clockwise direction resulting in net encirclements of the point $(-1 + j0)$ as nil indicating that the system is stable.

The system discussed above is stable only for a particular range of K any decrease or increase in the value of K makes the system unstable. Such systems are called conditionally stable systems.

The concepts of gain margin and phase margin are not applicable to conditionally stable systems.

CONSTANT - M-CIRCLES (MAGNITUDE)

The open-loop transfer function $G(s)$ of a unity feedback control system is a complex quantity.

$$G(s) = x + jy \quad , \quad H(s) = 1$$

$$s = j\omega ,$$

$$M = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

Magnitude = $M = \dfrac{x+jy}{1 + x + jy}$ —— ①

Taking modulus

$$|M| = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}} \text{ —— ②}$$

Squaring eqn ② and Simplifying

$$M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$$

$$\Rightarrow M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2(1+x^2+2x+y^2) = x^2+y^2$$

$$\Rightarrow M^2 + M^2x^2 + 2M^2x + M^2y^2 = x^2+y^2$$

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			1	2	3	4
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12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

$$\Rightarrow (1-M^2)x^2 - 2M^2x + (1-M^2)y^2 = M^2.$$

$$\Rightarrow x^2 - \left(\frac{2M^2}{1-M^2}\right)x + y^2 = \frac{M^2}{1-M^2} \quad \text{--- (3)}$$

Making perfect square adding $\left(\frac{M^2}{1-M^2}\right)^2$ to both sides to equ^n (3)

$$\Rightarrow x^2 - \frac{2M^2}{1-M^2}x + \left(\frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2}{1-M^2} + \left(\frac{M^2}{1-M^2}\right)^2$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2(1-M^2)+M^4}{(1-M^2)^2}$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2 \quad \text{--- (4)}$$

For different values of M, equ^n (4) represents a family of circles with centre at

$$\left(x = \frac{M^2}{1-M^2},\ y = 0\right).$$

and radius $\frac{M}{1-M^2}$

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8	9	10	11	12	13	14
15	16	17	18	19	20	21

For a particular circle the values of M (Magnitude of closed loop transfer function) is const, therefore, these circles are called const. M-circle.

Fig 7.11.1.

CONSTANT N-CIRCLE (PHASE ANGLES)

From eqn (1) the phase angle of the closed loop transfer function of a unity feedback control system is given by

$$\phi = \angle \frac{C(s)}{R(s)} = \angle \frac{x+jy}{1+x+jy} \quad \text{—— (5)}$$

The phase angle ϕ

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

Taking tan on both sides.

$$\tan\phi = \tan\left(\tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)\right)$$

$$= \frac{\tan\tan^{-1}\left(\frac{y}{x}\right) - \tan\tan^{-1}\frac{y}{1+x}}{1 + \tan\tan^{-1}\frac{y}{x} \cdot \tan\tan^{-1}\left(\frac{y}{1+x}\right)}$$

$$= \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y}{x} \times \frac{y}{1+x}}$$

$$\tan\phi = \frac{\frac{y(1+x) - yx}{(x(1+x))}}{\frac{x(1+x)+y^2}{(x(1+x))}}$$

$$\tan\phi = \frac{y}{x^2+x+y^2}$$

put $\tan\phi = N$

$$N = \frac{y}{x^2+x+y^2}$$

$$\Rightarrow x^2+x+y^2 = \frac{y}{N}$$

$$\Rightarrow x^2+x+y^2 - \frac{y}{N} = 0.$$

Making perfect square.

$$x^2 + 2\cdot x\cdot\frac{1}{2} + \frac{1}{4} - \frac{1}{4} + y^2 - 2\cdot\frac{y}{2N} + \frac{1}{4N^2} - \frac{1}{4N^2} = \frac{1}{4} + \frac{1}{4N^2}$$

$$= \left(x+\frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\frac{1}{4}+\frac{1}{4N^2}\right) \quad -(6)$$

for different values of N equn (6) represent a family of circle with centre at $x = -\frac{1}{2}$, $y = \frac{1}{2N}$.

$$r = \sqrt{\frac{1}{4}+\frac{1}{4N^2}}$$

M	Centre $x = \frac{M^2}{1-M^2}, y = 0$	Radius $r = \frac{M}{1-M^2}$
0.5	0.33	0.67
1.0	∞	∞*
1.2	−3.27	2.73
1.6	−1.64	1.03
2.0	−1.33	0.67
3.0	−1.13	0.38

*Intersection with real axis at $x = -0.5$.

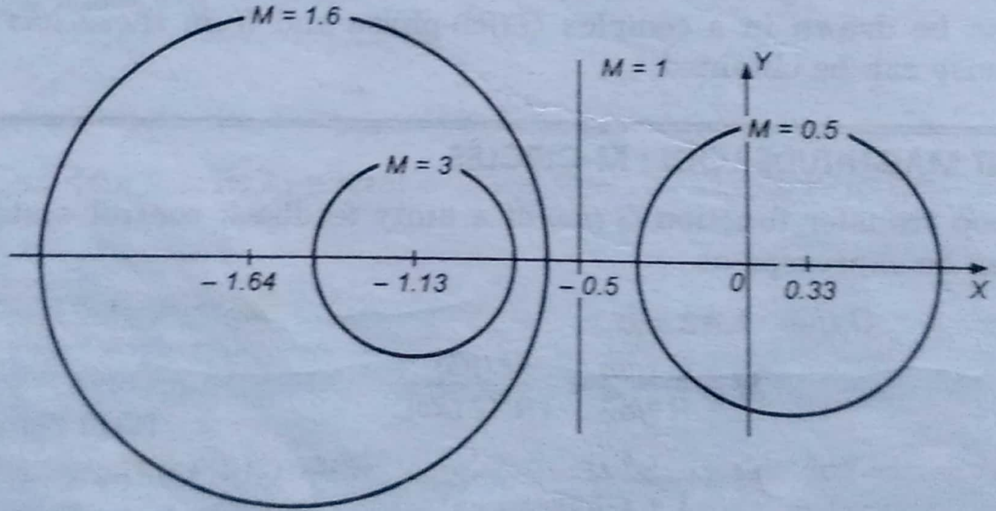

Fig. 7.11.1. *M*-circles.

In $G(j\omega)$ plane the Nyquist plot is superimposed on M-circle and the points of intersection give the magnitude of $C(j\omega)/R(j\omega)$ at different values of ω.

ϕ	$N = \tan\phi$	Centre $x = -\frac{1}{2}, y = \frac{1}{2N}$	Radius $R = \sqrt{\frac{1}{4} + \frac{1}{4N^2}}$
$-90°$	∞	0	0.5
$-60°$	-1.732	-0.289	0.577
$-50°$	-1.19	-0.42	0.656
$-30°$	-0.577	0.866	1.0
$-10°$	-0.176	-2.84	2.88
$0°$	0	∞	∞
$+10°$	0.176	2.84	2.88
$+30°$	0.577	0.866	1.0
$+50°$	0.19	0.42	0.656
$+60°$	1.732	0.289	0.577
$+90°$	∞	0	0.5

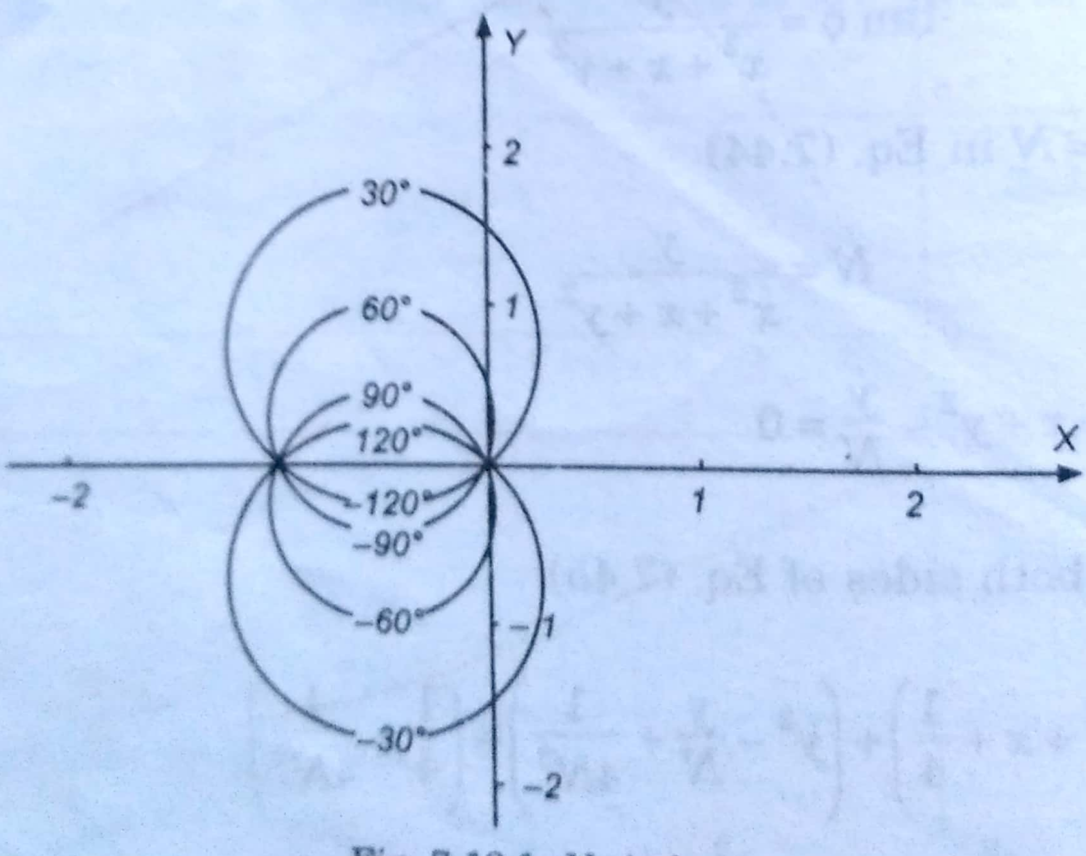

Fig. 7.12.1. *N*-circles.

7.15. NICHOLS CHART

The constant M and constant N circles in $G(j\omega)$ plane can be used for the analysis and design of control systems. However the constant M and constant N circles in gain phase plane *i.e.* graph having gain in decibel along the ordinate and phase angle along the abscissa are prepared for system design and analysis as these plots supply information with less manipulations. The M and N circles of $G(j\omega)$ in the gain phase plane are transformed into M and N contours in rectangular co-ordinates. A point on the constant M loci in $G(j\omega)$ plane is transferred to the gain phase plane by drawing the vector directed from the origin of $G(j\omega)$ plane to the particular point on the M circle and then measuring the length db, angle in and degree. These values of length and angles are the coordinates of the corresponding point in the gain phase plane as shown in Fig. 7.15.1 and 7.15.2. The critical point in $G(j\omega)$, plane corresponds to the point of zero decibel and – 180° in the gain-phase plane. Plot of M and N circles in gain phase plane is shown in Fig. 7.15.3 and known as 'Nichols chart'.

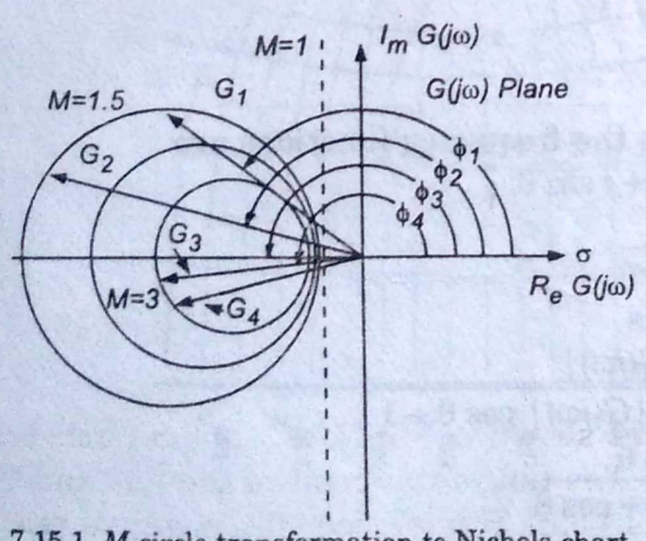

Fig. 7.15.1. *M*-circle transformation to Nichols chart.

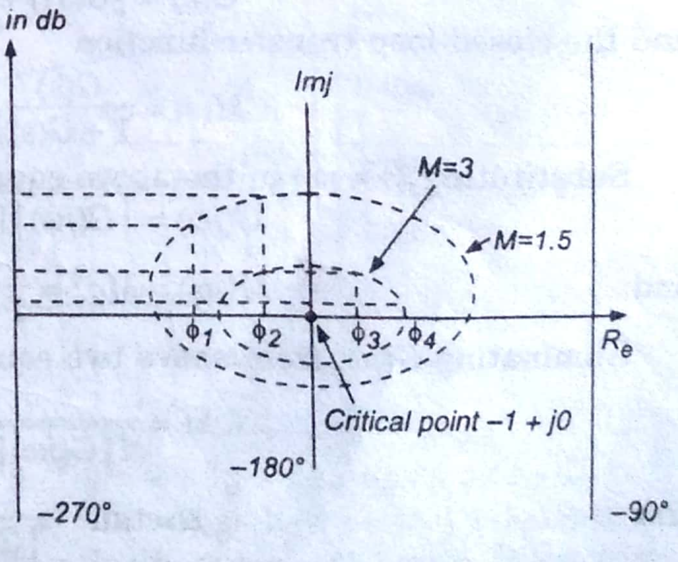

Fig. 7.15.2. *M*-circles in gain-phase plane.

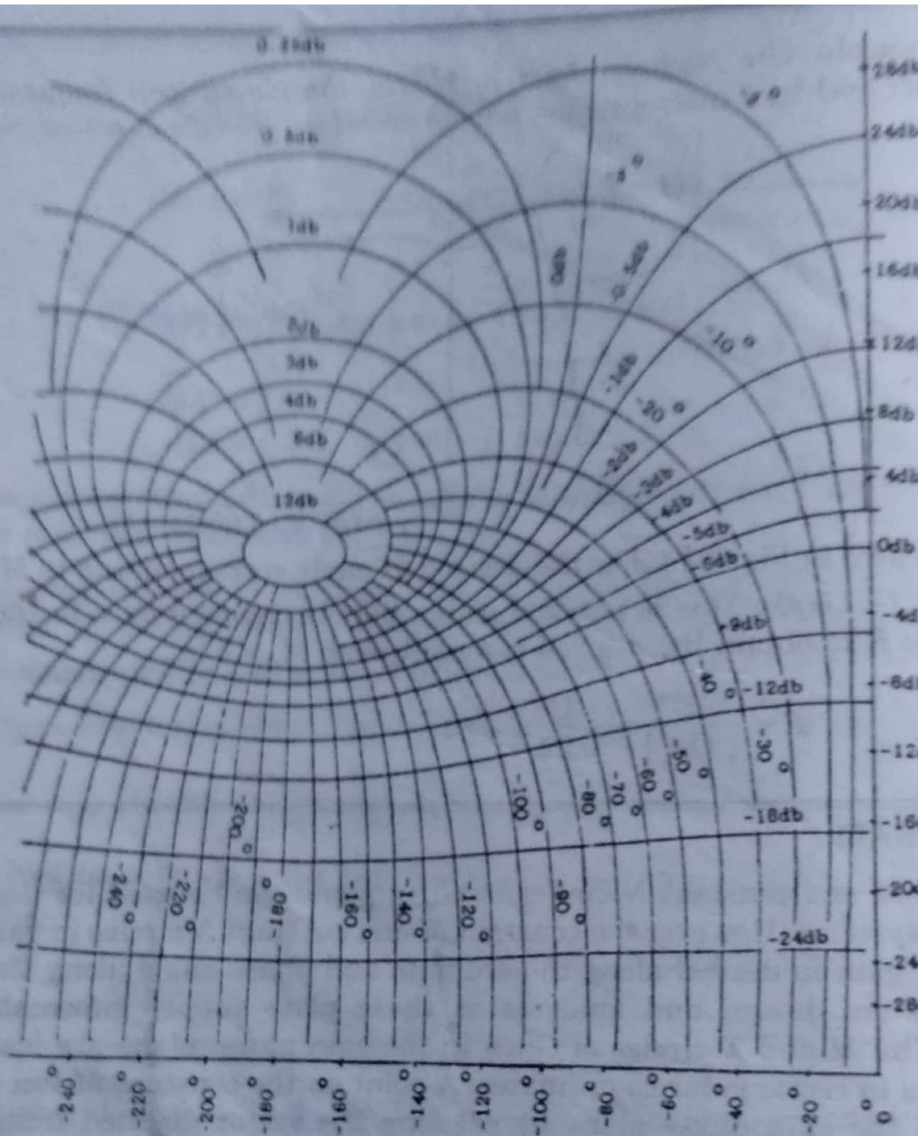

Fig. 7.15.3. Nichols chart.

Constant M and constant N circles in the Nichols chart are deformed into squashed circles. The complete Nichols chart extends for the phase angle of $G(j\omega)$ from 0 to $-360°$ but the region of $\angle Gj(\omega)$ generally used for analysis of systems is between $-90°$ and $-270°$. These curves repeat after every 180° interval. If the open-loop transfer function of the unity feedback system $G(s)$ is expressed as

$$G(s) = |G(s)|\, e^{j\theta} = |G(s)|\,[\cos\theta + j\sin\theta]$$

and the closed-loop transfer function

$$M(s) = \frac{G(s)}{1+G(s)}$$

Substituting ($s = j\omega$) in the above equations the frequency functions are

$$G(j\omega) = |G(j\omega)|\,[\cos\theta + j\sin\theta]$$

and
$$M(j\omega) = Me^{j\phi} = \frac{G(j\omega)}{1+G(j\omega)}$$

Eliminating $G(j\omega)$ from above two equations

$$M = \frac{|G(j\omega)|}{\sqrt{|G(j\omega)|^2 + 2|G(j\omega)|\cos\theta + 1}}$$

and
$$\phi = \tan^{-1}\frac{\sin\theta}{|G(j\omega)| + \cos\theta}$$

These equations define the plots in Nichols chart shown in Fig. 7.15.3.